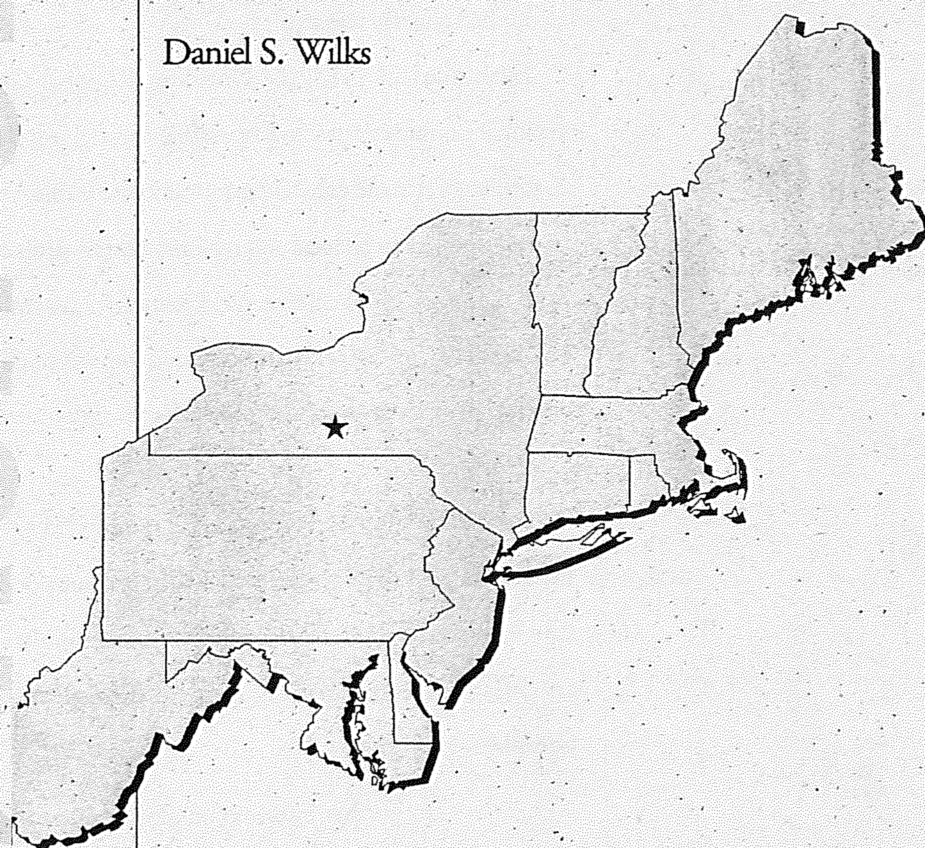
RESEARCH SERIES

NORTHEAST REGIONAL CLIMATE CENTER

Spline-Interpolated Parameters for Adjusting Climatological Precipitation Distributions using the 30- and 90-Day Outlooks

Daniel S. Wilks



Cornell University
Ithaca, New York
Publication No. RR 92-2
November 1992

The mission of the Northeast Regional Climate Center (NRCC) is to facilitate and enhance the collection, dissemination and use of climate data as well as to monitor and assess climatic conditions and impacts in the twelve-state, northeastern region of the United States. Implementing this mission involves three programmatic objectives: 1) the development and management of regional climate data bases, 2) the dissemination of information and educational services regarding climate and its impacts, and 3) the performance and support of applied climate research.

Established in 1983, the Northeast Regional Climate Center (NRCC) is one of six regional climate centers now operating throughout the nation. These regional centers serve as sources of climate data and information to public and private institutions and individuals as well as expertise on local and regional climate problems. The Center's staff cooperate with State Climatologists and research scientists in disseminating climate data and information, analyzing environmental and economic impacts of climate variability, and developing new applications of weather and climate data for agriculture, business, industry, and government operations.

The NRCC Research Report series is intended to make available to interested users the full results of climate research that has been supported by the NRCC. This report series supplements the normal reporting of research results in professional journals and provides an outlet for more complete and comprehensive accounts of work performed than is generally possible in journals.

For further information please write or call:

Northeast Regional Climate Center
1123 Bradfield Hall
Cornell University
Ithaca, New York 14853-1901
(607) 255-1751

The Northeast Regional Climate Center is supported by a Grant from the National Oceanic and Atmospheric Administration.

Spline-Interpolated Parameters for Adjusting Climatological Precipitation Distributions using the 30- and 90-Day Outlooks

Daniel S. Wilks
Department of Soil, Crop and
Atmospheric Sciences
Cornell University

Northeast Regional Climate Center
Research Series
Publication No. RR 92-2
November 1992

INTRODUCTION

Recently a method has been described (Wilks and Eggleston 1992) which allows adjustment of gamma distributions representing monthly or seasonal precipitation totals in a manner consistent with the 30- and 90-day precipitation outlooks issued by the Climate Analysis Center (CAC) (Epstein 1988, Wagner 1989). The approach is analogous to that of Lehman (1987), which adjusts the parameters of Gaussian distributions representing average temperatures on the basis of the outlook probabilities for temperatures. The motivation in both cases is to allow users to evaluate probabilities for outcome thresholds other than the two (the 30th and 70th percentiles) to which the outlooks directly pertain.

STRUCTURE OF THE OUTLOOKS

The procedure is implemented through the conventional assumption that monthly or seasonal precipitation totals follow a gamma distribution, the probability density function for which is

$$f(x;\alpha,\beta) = \frac{(x/\beta)^{\alpha-1}\exp(-x/\beta)}{\beta\,\Gamma(\alpha)} \quad ; x \geq 0; \alpha, \beta > 0 \ . \tag{1}$$

Here x is the random variable (total precipitation), α is the shape parameter, β is the scale parameter, and Γ denotes the gamma function. The function (1) is characterized by mean $\mu = \alpha\beta$ and variance $\sigma^2 = \alpha\beta^2$. It takes on a wide variety of shapes, ranging from strongly skewed and J-shaped for $\alpha \leq 1$ to approximately Gaussian for $\alpha \geq 50$. Using the gamma distribution is also convenient since monthly and seasonal precipitation are reported in terms of quantiles of climatological gamma distributions in the *Weekly Climate Bulletin* (Ropelewski et al. 1985).

The CAC outlooks divide the distribution of precipitation for a particular location and time period into three classes, separated by the 30th percentile (q_{30}) and the 70th percentile (q_{70}). These two points are given by

$$0.30 = \int_0^{q_{30}} f(x;\alpha_0,\beta_0)\,dx \quad , \tag{2}$$

and

$$0.70 = \int_0^{q_{70}} f(x;\alpha_0,\beta_0)\,dx \quad , \tag{3}$$

where the subscripts on the gamma distribution parameters denote that it is the unmodified climatological precipitation distribution being integrated. Precipitation outcomes smaller than q_{30} are denoted "dry," and outcomes larger than q_{70} are denoted "wet." Precipitation totals between these two quantiles are regarded as being "near normal."

The CAC outlooks specify a probability that the precipitation in the upcoming period will fall into either the "wet" or "dry" category, with the probability of "near-normal" always fixed at 0.4. The convention is adopted here and in Wilks and Eggleston (1992) that the forecast probability, z, pertains to the dry outcome. Thus the full forecast implied by a particular probability z is

$$\begin{aligned} \Pr(0 \le x < q_{30}) &= z \\ \Pr(q_{30} \le x < q_{70}) &= 0.4 \\ \Pr(q_{70} \le x) &= 0.6 - z \end{aligned} \tag{4}$$

MODIFYING THE CLIMATOLOGICAL GAMMA DISTRIBUTION

The procedure is implemented by modifying the climatological distribution parameters α_0 and β_0 in a manner consistent with the outlook probability for precipitation below q_{30}. This is most conveniently done using the ratio of means, $\mu^*/\mu_0 = (\alpha^*\beta^*)/(\alpha_0\beta_0)$, and the ratio of scale parameters, β^*/β_0, where starred symbols pertain to the modified distribution. These two ratios are related algebraically to the forecast z according to

$$\frac{\mu^*}{\mu_0} = 1 + b_M (z - 0.3) \tag{5}$$

and

$$\frac{\beta^*}{\beta_0} = 1 + b_1 (z - .3) + b_2 (z - 0.3)^2 \tag{6}$$

(Wilks and Eggleston 1992), which ensure that the modified and climatological distributions are identical when the climatological forecast z=0.30 is issued.

The three regression parameters b_M, b_1, and b_2 in (5) and (6) vary as a function of the climatological shape parameter α_0 as shown in Figure 1. Use of this figure may be inconvenient if the procedure is to be used frequently, or is to be automated. Unfortunately, the relationships portrayed do not appear to have simple parametric representations. A practical alternative is to use cubic spline interpolation (e.g., Press et al. 1986) from the set of values presented in Table 1. This is accomplished implementing, for each of the three coefficients b in (5) and (6),

$$\hat{b} = A b_j + (1 - A) b_{j+1} + C b_j'' + D b''_{j+1} \quad ; \tag{7}$$

where

$$A = \frac{\alpha_{j+1} - \alpha_0}{\Delta \alpha_j} \quad , \qquad \alpha_j \le \alpha_0 \le \alpha_{j+1} \quad , \tag{8}$$

$$C = \frac{A^3 - A}{6} (\Delta\alpha_j)^2 \tag{9}$$

and

$$D = -\frac{A^3 - 3A^2 + 2A}{6} (\Delta\alpha_j)^2 \quad . \tag{10}$$

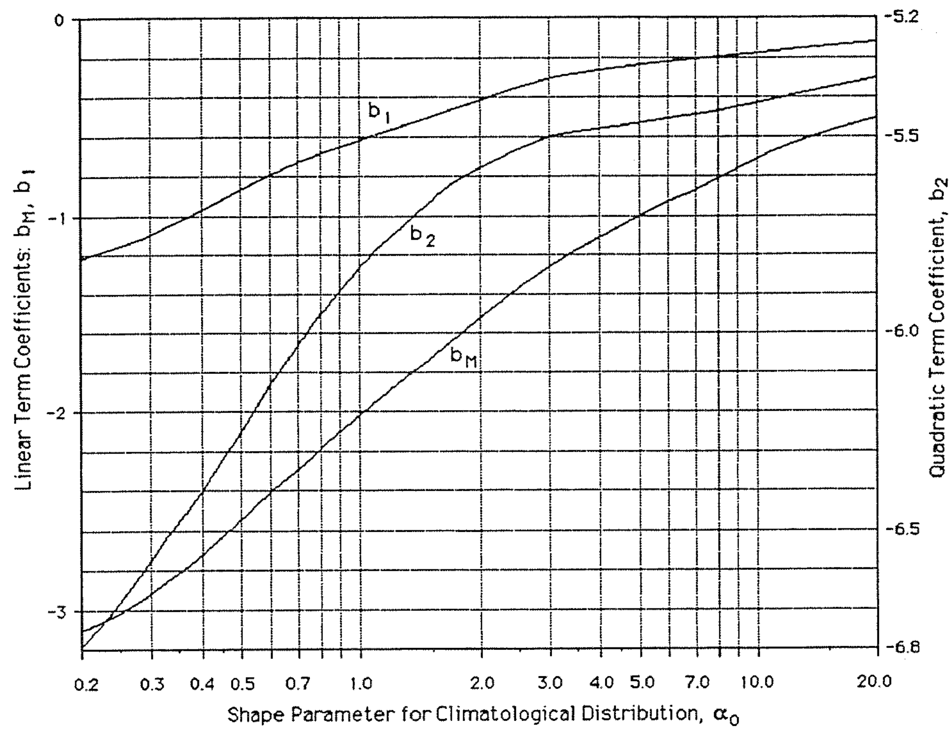


Figure 1. Dependence of the three parameters used in (5) and (6) on the shape parameter of the underlying climatological distribution, α_0. Curves obtained using the spline interpolation algorithm (7) - (10) and the data in Table 1.

Table 1. Parameters for cubic spline interpolation of the parameters b_M, b_1, and b_2 used in (5) and (6). Interpolation algorithm is given in (7) - (10).

j	α_j	$\Delta\alpha_j$	b_M	b''_M	b_1	b''_1	b_2	b''_2
1	0.1	0.1	-3.307	0.0000	-1.316	0.0000	-7.053	0.0000
2	0.2	0.3	-3.110	0.2619	-1.209	1.459	-6.795	-4.833
3	0.5	0.5	-2.553	-2.965	-0.870	-2.690	-6.252	-2.513
4	1.0	1.0	-2.035	-0.5169	-0.630	-0.0667	-5.833	-0.7241
5	2.0	3.0	-1.534	-0.1767	-0.425	-0.1047	-5.579	-0.0754
6	5.0	5.0	-1.010	-0.0091	-0.254	0.0054	-5.472	0.0056
7	10.0	10.0	-0.723	-0.0057	-0.172	-0.0033	-5.415	-0.0019
8	20.0	30.0	-0.515	-0.0004	-0.124	0.0002	-5.536	-0.0003
9	50.0	—	-0.327	0.0000	-0.074	0.0000	-5.391	0.0000

EXAMPLE

The procedure will be illustrated for the case of January total precipitation at Ithaca, New York. The available period of record is the 112 years 1879-1990, for which the fitted gamma distribution has $\alpha_0 = 4.41$ and $\beta_0 = 1.16$ cm. The parameters b_M, b_1 and b_2 are computed from Table 1 and (7)-(10). In this case, since $2.0 < \alpha_0 < 5.0$ (cf. equation (8)), j=5. Then,

$$A = (5.0-4.41)/3.0 = 0.1967,\ C = -0.2836, \text{ and } D = -0.4274.$$

Equation (7) then gives

$$b_M = A(-1.534) + (1-A)(-1.010) + C(-0.1767) + D(-0.0091) = -1.059.$$

Similarly, $b_1 = -0.260$ and $b_2 = -5.474$.

Consider the two (unusually confident) forecasts z = 0.5 and z = 0.1. In the former ("dry" forecast) case, using (6),

$$\beta^* = 1.16 \text{ cm } [\ 1 + (-0.260)(0.5-.3) + (-5.474)(0.5-.3)^2\ ] = 0.85 \text{ cm}.$$

Using (5),

$$\alpha^* = (\alpha_0)\ (\beta^*/\beta_0)^{-1}\ (\mu^*/\mu_0) = (4.41/0.729)\ [\ 1 + (-1.059)(0.5-.3)] = 4.77.$$

Similarly, for the "wet" forecast z = 0.1, $\beta^* = 0.97$ cm and $\alpha^* = 6.41$. These two adjusted gamma distributions are shown in Figure 4 of Wilks and Eggleston (1992).

REFERENCES

Epstein, E.S., 1988. Long-range weather prediction: limits of predictability and beyond. *Weather and Forecasting*, 3, 69-75.

Lehman, R.L., 1987. A model for decision making based on NWS monthly temperature outlooks. *Journal of Climate and Applied Meteorology*, 26, 263-274.

Press, W.H., B.P. Flannery, S.A. Teukolsky, and W.T. Vettering, 1986. *Numerical Recipes*. Cambridge University Press, 818 pp.

Ropelewski, C.F., J.E. Janowiak, and M.S. Halpert, 1985. The analysis and display of real time surface climate data. *Monthly Weather Review*, 113, 1101-1106.

Wagner, A.J., 1989. Medium- and long-range forecasting. *Weather and Forecasting*, 4, 413-426.

Wilks, D.S., and K.L. Eggleston, 1992. Estimating monthly and seasonal precipitation distributions using the 30- and 90-day outlooks. *Journal of Climate*, 5, 252-259.

NOTES

NRCC RESEARCH SERIES

Knapp, W.W. and K.L. Eggleston, *Some Impacts of Recent Climate Variability on the Northeast,* NRCC Research Publication RR 91-1.

Wilks, D.S., *Gamma Distribution Probability Tables for Use in Climatology,* NRCC Research Publication RR 91-2.

Wilks, D.S., *Spline Interpolated Parameters for Adjusting Climatological Precipitation Distributions using the 30- and 90-Day Outlooks,* NRCC Research Publication RR 92-2.

NRCC DIGITAL DATA SETS

Eggleston, K.L. and D.S. Wilks, *Gridded Monthly Precipitation Distribution Parameters for the Continental United States,* NRCC Data Set DS 92-1.

CORNELL
UNIVERSITY
Department of Soil, Crop and Atmospheric Sciences
Ithaca, New York 14853